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6c. ADDRESS (City, State, and ZIP Code)
Tucson, AZ 85721

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Bldg 410
Bolling AFB DC 20332-6448

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Dr. Newell

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Dr. Newell and his co-workers published 40 papers during the period of the support! They were predominantly in the areas of nonlinear optics and fluid turberlence. Dr. Newell and his colleagues are world leaders in the analysis of the type of nonlinear PDE which regularly occur in these two physical areas. Moreover, 3 PhD students were produced and many visitors/postdocs were supported who could interact with the University of Arizona researchers. The growing recognition of the University of Arizona group has well repaid the Air Force investment.

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22b. TELEPHONE (Include Area Code)
(202) 767-5027

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NM

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ARIZONA CENTER FOR MATHEMATICAL SCIENCES

Quarterly Report

Department of Mathematics
University of Arizona
Tucson, Arizona 85721

September 1987

Sponsored by the Air Force Office of Scientific Research under the
University Research Initiative
Grant No. FQ8671-8601551

THE CENTER

The primary goal of the Center is to provide an environment for research and learning in the Mathematical Sciences. The basic themes of this research are the understanding and application of nonlinear processes with particular emphases on optics and fluids. The Center will be strongly interdisciplinary and each year will host graduate students, post- doctoral fellows, long and short-term visitors and several workshops. Some of the workshops will be padagogic in nature and address fairly broad issues of topical interest (e.g. turbulence, artificial intelligence, wave propagation, nonlinear numerical analysis). Others will be devoted to 'state of the art' discussions of specific problems (e.g. nonlinear waveguides, Anderson localization). It is also expected that many colleagues from other universities, national laboratories and Air Force research centers will visit the Center at regular intervals. There is funding to support graduate students, post-doctoral fellows and visiting faculty.

We have had a very successful first year. There were four workshops which stimulated a great deal of collaboration both among members of the Center and with colleagues at other universities, and many short term visitors. In addition, we have enjoyed interacting with colleagues from Kirtland AFB and have discovered some areas of mutual interest. Information on a one day workshop held at Kirtland on August 24, 1987 follows. An announcement of a workshop on *Singularities in PDE's,* which will be jointly sponsored by NSF and AFOSR, is given just before the appendices. New faculty members at Arizona who will also be members of the Center are: Bruce Bayly, Moysey Brio, David Levermore and Brenton Le Mesurier.

PERSONNEL

Co-Directors

Alan C. Newell	(602/621-2868)
David W. McLaughlin	(602/621-4664)

Center Faculty

Gregory Baker	(602/621-6892)
Nicholas Ercolani	(602/621-4763)
Paul Fife	(602/621-6869)
Hermann Flaschka	(602/621-6862)
Christopher Jones	(602/621-6892)
Jerry Moloney	(602/621-6652)
Charles Newman	(602/621-6861)
David Rand	(602/621-4664)
Tudor Ratiu	(602/621-4397)
Maciej Wojtkowski	(602/621-6695)
Lai-Sang Young	(602/621-6892)

Center Staff

Administrative Assistant: Robert Ray	(602/621-4361)
Computer Software Specialist: Robert Indik	(602/621-4599)
Computer Operator: David Hamara	(602/621-4717)

Post-Doctoral Fellows and Visiting Faculty Closely Associated with the Center

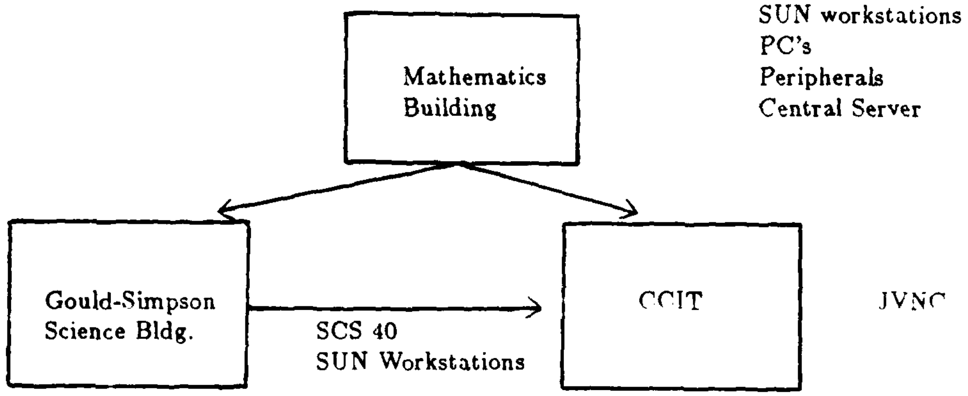
Douglas Abraham, Ph.D., 1968, King's College (Statistical Mechanics)
Wayne Arter, Ph.D., 1983, Trinity College, Cambridge (Computational Science)
David Barsky, Ph.D., 1987, Rutgers University (Statistical Mechanics)
Andrew Bernoff, Ph.D., 1985, Trinity College, Cambridge (Nonlinear Dynamics)
Jean-Guy Caputo, Ph.D., 1986, University of Grenoble (Dynamical Systems)
Martin Casdagli, Ph.D., 1985, Warwick University, England (Dynamics)
Patrick Dunne, Ph.D., 1987, M.I.T. (Hydrodynamic Stability, Nonlinear Waves)
G. R. Grimmett, Ph.D., 1974, Mathematical Institute, Oxford (Probability)
Robert Maier, Ph.D., 1983, Rutgers University (Computer Science)
Alistair Mees, Ph.D., 1973, Cambridge University (Dynamical Systems)
Edward Overman, II, Ph.D., 1978, Ohio State University (Computational Science)
Thierry Passot, Ph.D., 1987, Observatoire de Nice (Turbulence, Painlevé Analysis)
Ron Sawatzky, Ph.D., 1987, University of Alberta (Wave Propagation in Cont. Media)
M'Hamed Souli, Ph.D., 1984, Universite de Nice (Computational Science)
Henryk Zoladek, Ph.D., 1983, Moscow State University (Bifurcation Theory)

COMPUTER FACILITIES

Sophisticated computer workstations, peripherals, and installation of communications links to various computers allows access to a fully interactive, large-scale, and graphics computing environment essential for calculations and processing of experimental data to pursue these investigations.

The ETHERNET system links the CCIT (Center for Computing and Information Technology and the JVNC) to the Gould-Simpson Building (the SCS-40) and to the Mathematics Building (SUN workstations, PC's, peripherals) where the SUNs are also connected by ETHERNET to a central server connecting to the SCS-40 and the JVNC with a high speed communications link.

ETHERNET NETWORK



KIRTLAND

On August 24 a group from the University of Arizona, led by Dr. Alan Newell, participated in a one day workshop hosted by the Center for Nonlinear Optics at Kirtland AFB, New Mexico. **Those participating in the workshop were:**

Mr. A. Aceves
University of Arizona

Mr. H. Adachihara
University of Arizona

Dr. C. Clayton
Kirtland AFB

Capt. A. Corvo
Kirtland AFB

Dr. C. Dent
GCD Associates

Dr. T. Gavrielides
Kirtland AFB

Dr. B. LeMesurier
University of Arizona

Dr. D. W. McLaughlin
University of Arizona

Dr. J. V. Moloney
University of Arizona

Dr. A. C. Newell
University of Arizona

Dr. P. Peterson
Kirtland AFB

Dr. M. Souli
University of Arizona

Dr. R. Sawatzky
University of Arizona

Dr. L. Schelonka
Kirtland AFB

Papers were presented by:

Mr. A. Aceves
Snell's Laws for Nonlinear Dielectrics

Capt. A. Corvo
Overview of the Quantum Optics Branch

Dr. B. LeMesurier
Focusing and Multifocusing Solutions of the Nonlinear Schrödinger Equations

Dr. L. Schelonka
The Fidelity of Stimulated Brillouin Scattering with Weak Aberrations

Mr. H. Adachihara
Transverse Instability in the Optical Ring Cavity

Dr. C. Dent
Chaos in Diode Lasers

Dr. P. Peterson
Optical Parameter Amplification

WORKSHOPS

NUMERICAL SOLUTIONS OF NONLINEAR DIFFERENTIAL EQUATIONS
Organizers: A. Iserles, D. W. McLaughlin, A. C. Newell
January 12 - 15, 1987

Recent advances with new algorithms and important theoretical advances, as well as the availability of supercomputers, which are transforming numerical analysis of nonlinear differential equations were presented and discussed.

Approximate Cost of Workshop: $8,060.

RANDOM SCHRÖDINGER EQUATIONS
Organizers: W. Faris, J-G. Caputo, R. Maier
February 19 - 21, 1987

Directions for research that may eventually lead to an understanding of conservative linear wave propagation in random media, including localization and the presence of nonlinearities were explored and discussed.

Approximate Cost of Workshop: $11,200.

CURRENT IDEAS IN NONLINEAR SCIENCE: FIRST ANNUAL WORKSHOP FOR ADVANCED UNDERGRADUATES
Organizers: D. W. McLaughlin, A. C. Newell
March 1 - 3, 1987

The workshop communicated to potential graduate students the present level of excitement and activity in the areas of Nonlinear Waves; Dynamical Systems; Geometry, Topology and Nonlinearity and Statistical Physics.

Approximate Cost of Workshop: $5,312.

STATE OF THE ART DEVELOPMENTS IN NONLINEAR OPTICS
Organizers: J. Moloney, A. C. Newell
March 29 - April 4, 1987

A forum was provided for discussion between physicists working on rapidly developing areas of nonlinear optics, and mathematicians familiar with a broad variety of phenomena in nonlinear waves, dynamical systems and computational methods.

Approximate Cost of Workshop: $12,450.

Proposed Workshop

On

Singularities In Nonlinear Partial Differential Equations

Organizers: N. Ercolani, A. C. Newell

March 12 - 20, 1988

Introduction

There are two branches of mathematics in which the study of singularities has been very relevant. One is *nonlinear evolution equations* which arise in fluid mechanics, plasma physics, condensed matter physics and reaction diffusion chemistry. The other is *differential geometry* and, in particular, the areas of minimal surface theory, harmonic maps, conformal deformations of Riemannian metrics and general relativity. For those equations which pertain to these subjects, singularities often reveal crucial facets of the system which they model.

There has been much recent activity in the analysis of singularities coming from both physics and geometry. In light of this it seems timely to have experts from both communities engage in a dialogue which would encourage the exchange of ideas and techniques. For this purpose the Department of Mathematics at the University of Arizona proposes to host a week long conference in March 1988 on *Singularities in Nonlinear Differential Equations.*

Participants

To date we have contacted a number of leading mathematicians and scientists, each of whom has agreed, barring unexpected circumstances, to participate.

M. Atiyah
Oxford University

A. Chorin
UC Berkeley

R. DiPerna
UC Berkeley

R. Kohn
Courant Institute

P. Kronheimer
Oxford University

A. Majda
Princeton University

J. Marsden
UC Berkeley

G. Papanicolau
Courant Institute

J. Polking
Rice University

R. Schoen
UC San Diego

E. Siggia
Cornell University

L. Simon
Stanford University

M. Weinstein
Princeton University

S. T. Yau
UC San Diego

Organization

The organization of the workshop will be similar to that of the recent successful workshops in dynamical systems, numerical analysis, and mathematical physics held at Arizona during the past few years. We plan to advertise broadly. In addition to the above mentioned participants, we will seek interested colleagues at the post doctoral and young assistant professor levels. We plan to invite approximately forty participants.

For additional information on this workshop, please contact:

Robert Ray
University of Arizona
Arizona Center for Mathematical Sciences
Gould-Simpson 815
Tucson, AZ 85721 USA

APPENDICES

I.	Workshop on Numerical Solutions of Nonlinear Differential Equations
II.	Workshop on Random Schrödinger Equations
III.	Workshop on Current Ideas in Nonlinear Science
IV.	Workshop on State of the Art Developments in Nonlinear Optics
V.	List of Preprints and Reprints with Abstracts

ANTICIPATED WORKSHOPS 1987 - 1988

I. Workshop on Singularities in Nonlinear Differential Equations
Organizers: N. Ercolani, A. C. Newell
March 12 - 20, 1988

II. Workshop on Lagrangian Turbulence
Organizer: M. Tabor, A. C. Newell
April 6 - 9, 1988

III. Workshop on Computational Sciences
Organizer: G. Baker, A. C. Newell
Dates have not yet been finalized.

IV. Workshop on Applied Probability
Organizer: C. Newman, A. C. Newell
Dates have not yet been finalized.

Workshop on Numerical Solutions of Nonlinear Differential Equations

Organizers: A. Iserles, D. W. McLaughlin, A. C. Newell

January 12 - 15, 1987

Recent advances with new algorithms and important theoretical advances as well as the availability of supercomputers, which are transforming numerical analysis of nonlinear differential equations were presented and discussed.

Approximate Cost of Workshop

Consulting fees	\$7,560
Food	500
Total	\$8,060

Workshop Participants

Wayne Arter
Culham & University of Arizona

Andy Bernoff
University of Arizona

Dave Broomhead
RSRE Malvern

Moysey Brio
Courant Institute

Jean Guy Caputo
University of Arizona

Martin Casdagli
University of Arizona

Y.F. Cheng
Claremont

George Corliss
Marquette

Phil Davis
Brown University

Clint Dawson
Rice University

Bjorn Engquist
UCLA

William Faris
University of Arizona

Herman Fasel
University of Arizona

Randy LeVeque
Seattle

Zheng Lin
University of Arizona

Iris Mack
MIT

D. W. McLaughlin
University of Arizona

Hans Mittelman
Arizona State University

N. Mittwollen
Braunschweig

Bill Morton
Oxford

Alan C. Newell
University of Arizona

Charles Newman
University of Arizona

Sheng Qin
Cambridge

David Rand
Warwick & University of Arizona

Dave Ruch
NMSU

Richard Sanders
Houston

Alan Feldstein
Arizona State University

Bengt Fornberg
Exxon

Wojciech Golich
NMSU

Thomas Hagstrom
SUNY

J. B. Herbst
Orange Free State

Arieh Iserles
Cambridge & University of Arizona

George Karniadakis
MIT

Benigno Lazaro
USC

Don Schwendeman
CalTech

Ricardo Soto
NMSU

Mike Tabor
Columbia University

J. W. Thomas
Colorado State University

Thomas Vincent
University of Arizona

Andre Weideman
MIT

Mary Wheeler
Rice University & Houston

Zhi-Xue Xu
University of Arizona

Workshop on Random Schrödinger Equations

Organizers: W. Faris, J-G. Caputo, R. Maier

February 19 - 21, 1987

Directions for research that may eventually lead to an understanding of conservative wave propagation in random media, including localization and the presence of nonlinearities were explored and discussed.

Approximate Cost of Workshop

Consulting fees	\$10,850
Food	350
Total	\$11,200

Workshop Participants

Eric Akkermans
Grenoble

Serge Aubry
Saclay/Los Alamos

Alan Bishop
Los Alamos

Anton Bovier
UC Irvine

Jean Guy Caputo
University of Arizona

René Carmona
UC Irvine

Massimo Campanino
UC Irvine

Martin Casdagli
University of Arizona

Morrel H. Cohen
Exxon

Walter Craig
Stanford

Francois Delyon
École Polytechnique

Charles Doering
Los Alamos

B. Doucot
AT&T Bell Labs

Barry Ganapol
University of Arizona

Hyatt Gibbs
University of Arizona

Leon Glass
McGill

Kenneth Golden
Rutgers University

W. Martin Greenlee
University of Arizona

Richard Griego
University of New Mexico

W. Horton
UT Austin

James Howland
University of Virginia

Russell Johnson
USC

Abel Klein
UC Irvine

Robert Knapp
Courant Institute

Willis E. Lamb, Jr.
University of Arizona

David Levermore
Lawrence Livermore Laboratory

William Faris
University of Arizona

Ronald Fisch
Washington University

David W. McLaughlin
University of Arizona

Alan C. Newell
University of Arizona

Charles Newman
University of Arizona

George Papanicolau
Courant Institute

J. F. Perez
UC Irvine

Jürgen Pöschl
Cornell University

R. Rammal
AT&T Bell Labs

Harvey A. Rose
Los Alamos

David Russell
Los Alamos

Alwyn Scott
University of Arizona

Mike Shelley
Princeton University

Robert Maier
University of Arizona

J. M. Maillard
Université Pierre et Marie Curie

John Sipe
University of Arizona

Bernard Souillard
École Polytechnique

A. Speis
UC Irvine

Thomas Spencer
Institute for Advanced Study

Lawrence Thomas
University of Virginia

David Thouless
University of Washington

Michael Tratnik
Toronto

Michael Weinstein
Princeton University

Art Winfree
University of Arizona

Lai-sang Young
University of Arizona

Y. Q. Yin
University of Arizona

APPENDIX III

Current Ideas in Nonlinear Science: First Annual Workshop for Undergraduates

Organizers: D. W. McLaughlin, A. C. Newell

March 1 - 3, 1987

The workshop communicated to potential graduate students the present level of excitement and activity in the areas of Nonlinear Waves; Dynamical Systems; Geometry, Topology and Nonlinearity and Statistical Physics.

Approximate Cost of Workshop

Consulting fees	$3,450
Food	350
Lodging	$1,512
Total	$5,312

Workshop Participants

Kelly Alvey
University of Oregon

Evan Ayala
New York University

Tim Barclay
Rocky Mountain College

Andrew Beckwith
University of Oregon

Richard Braun
Santa Clara University

Brian Combs
Reed College

David Graser
Harvey Mudd College

Michael Holst
Colorado State University

Heather Hulett
University of Kansas

Aaron Klebanoff
UC Irvine

Philip Ma
University of Waterloo

Douglas Mar
University of Washington

Usha Ramgulam
University of Waterloo

Lori Smellegar
Stockton State College

Fred Taverner
UC Davis

Spiros Tsaltas
University of Waterloo

John Tucker
UC Irvine

Siu Wai Yiu
Ohio State University

Workshop on State of the Art Developments in Nonlinear Optics

Organizers: J. V. Moloney, A. C. Newell

March 29 - April 4, 1987

A forum was provided for discussion between physicists working on rapidly developing areas of nonlinear optics, and mathematicians familiar with a broad variety of phenomena in nonlinear waves, dynamical systems and computational methods.

Approximate Cost of Workshop

Consulting fees	$11,250
Food	1,200
Total	$12,450

Workshop Participants

F. T. Arecchi
Instituto Nacional de Ottica

A. Aceves
University of Arizona

I. Bigio
Los Alamos

L. W. Casperson
Portland State University

M. Cohen
NMSU

K. Druhl
Maharishi International University

R. Enns
Simon Fraser University

W. J. Firth
University of Strathclyde

J. A. fleck
Lawrence Livermore Laboratory

T. Gavrielides
Kirtland AFB

H. M. Gibbs
University of Arizona

J. P. Gordon
AT&T Bell Labs

H. Haus
MIT

D. Kaup
Clarkson University

Y. Kodama
Nagoya University

J. F. Lam
Hughes

W. E. Lamb, Jr.
University of Arizona

M. Lax
City College of the CUNY

B. LeMesurier
Rensselaer Polytech Institute

A. A. Maraduden
UC Irvine

F. Mattar
New York University

P. Meystre
University of Arizona

J. V. Moloney
Heriot-Watt

D. W. McLaughlin
University of Arizona

A. Nachman
AFOSR

A. C. Newell
University of Arizona

K. Ikeda
Kyoto University

C. Jones
University of Maryland

A. E. Kaplan
John Hopkins University

W. Kath
Northwestern University

C. Tourenne
Maharishi International University

H. Winful
University of Michigan

Y. Silberberg
Bell Communications Research

G. I. Stegeman
University of Arizona

R. H. Stolen
AT&T Bell Labs

W. J. Tomlinson
AT&T Bell Labs

R. G. Wenzel
Los Alamos

P. Yeh
Rockwell International

APPENDIX V

Preprints and Reprints

ACMS Publication 86-1
J-G. Caputo, W. Faris, A. C. Newell, C. M. Newman
Nonlinear Tunneling Through Random Media. Preprint.
ACMS Publication 86-2
A. C. Newell
Chaos and Turbulence. Part of Proceedings.
ACMS Publication 86-3
A. C. Newell
Chaos and Turbulence: Is There a Connection ? Siam.
ACMS Publication 86-4
A. C. Newell, M. Tabor, Y. B. Zeng
A Unified Approach to Painlevé Expansions. To appear in *Phys. D.*
ACMS Publication 86-5
J. D. Gibbon, A. C. Newell, M. Tabor, Y. B. Zeng
Lax Pairs, Bäcklund Transformations and Special Solutions for Ordinary Differential Equations. To appear in *Phys. D.*
ACMS Publication 86-6
A. C. Newell, Z. Yunbo
The Hirota Conditions. J. Math Physics 27, 2016 (1986).
ACMS Publication 86-7
L. Chierchia, N. Ercolani, D. W. McLaughlin
On the Weak Limit of Rapidly Oscillating Waves.
To appear in *Duke Math J.*
ACMS Publication 86-8
N. Ercolani, M. G. Forest, D. W. McLaughlin, R. Montgomery
Hamiltonian Structure for the Modulation Equations of a Sine-Gordon Wavetrain.
To appear in *Duke Math J.*
ACMS Publication 86-9
A. Mazor, A. R. Bishop, D. W. McLaughlin
Phase-Pulling and Space-Time Complexity in an AC Driven Damped One-Dimensional Sine-Gordon System. To appear in *Phys. Lett.*
ACMS Publication 86-10
A. Bishop, D. W. McLaughlin, E. A. Overman II
A Quasi-Periodic Route to Chaos in a Near-Integrable Partial Differential Equation: Homoclinic Crossings. Submitted to *Phys. Lett.*
ACMS Publication 86-11
A. R. Bishop, M. G. Forest, D. W. McLaughlin, E. A. Overman II
A Quasi-Periodic Route to Chaos in a Near-Integrable PDE. Physica D, 293-328 (1986).
ACMS Publication 86-12
N. Ercolani, D. W. McLaughlin, M. G. Forest
Homoclinic Orbits for the Periodic Sine-Gordon Equation.
Submitted to *Physica D.*
ACMS Publication 86-13
N. Ercolani, D. W. McLaughlin, M. G. Forest
Geometry of the Modulational Instability: Part I. Local Results, Part II. Global Results.
To be submitted to *Comm. Pure Appl. Math.*
ACMS Publication 86-14
H. Adachihara, D. W. McLaughlin, J. V. Moloney, A. C. Newell
Solitary Waves as Fixed Points of Infinite-Dimensional Maps for an Optical Biostable Ring Cavity: Analysis. To appear in *J. Math. Phys.*

ACMS Publication 86-15

A. Aceves, H. Adachihara, C. Jones, J. C. Lerman, D. W. McLaughlin, J. V. Moloney, A. C. Newell, *Chaos and Coherent Structure in Partial Differential Equations.* *Physica 18D, 85-112 (1986.)*

ACMS Publication 86-16

A. Aceves, J. V. Moloney, A. C. Newell
Trajectories of Surface Waves in Dielectrics at Nonlinear Interfaces
Preprint.

ACMS Publication 86-17

C. M. Newman
Another Critical Exponent Inequality for Percolation: $\beta \geq s/\delta$.
To appear in *J. Stat. Phys.*

ACMS Publication 86-18

H. R. Brand, P. S. Lomdahl, A. C. Newell
Evolution of the Order Parameter in Situations with Broken Rotational Symmetry.
Phys. Lett. A., 118-2, 67-73 (1986).

ACMS Publication 87-1

M. I. Aksman, E. A. Novikov
Metamorphases of Three-Dimensional Vortex Structures. Preprint.

ACMS Publication 87-2

M. Casdagli
Rotational Chaos in Dissipative Systems. In preparation.

ACMS Publication 87-3

M. Casdagli, J. M. Greene
Lack of Scaling for Break Up of K.A.M. Tori with Rotational Number a Cubic Irrational.
In preparation

ACMS Publication 87-4

M. Casdagli, D. A. Rand
Fluctuation Spectra for Time-Averages, Characteristic Exponents and Rotation Numbers.
In Preparation

ACMS Publication 87-5

A. Iserles, S. P. Nørsett
Order Stars and Rational Approximants to exp (2). To appear in *App. Numerical Analysis,* (special issue).

ACMS Publication 87-6

A. Iserles, S. P. Nørsett
Zeros of Transformed Polynomials. In preparation.

ACMS Publication 87-7

Charles M. Newman
Memory Capacity in Neural Network Models: Rigorous Lower Bounds.
Submitted to *Biol. Cybern.*

ACMS Publication 87-8

Andrew J. Bernoff
Slowly Varying Fully Nonlinear Wavetrains in the Ginzburg-Landau Equation. Preprint.

ACMS Publication 87-9

Q. Zou, E. A. Overman II, H. M. Wu, N. J. Zabusky
Contour Dynamics for the Euler Equations: Curvature Controlled Initial Node Placement and Accuracy. To appear in *Jour. Comp. Phys.*

ACMS Publication 87-10

N. Simányi, M. P. Wojtkowski
Two-Particle Billiard System with Arbitrary Mass Ratio.
To appear *New Dir. in Dyn. Sys.*

ACMS Publication 87-11

Maciej P. Wojtkowski
Bounded Geodesics for the Atiyah-Hitchin Metric. Preprint.

ACMS Publication 87-12

G. Caginalp, P. C. Fife

Dynamics of Layered Interfaces Arising from Phase Boundaries.

To appear in *Siam.*

ACMS Publication 87-13

M. Aizenman, J. T. Chayes, L. Chayes, C. M. Newman

Discontinuity of the Magnetization in One-Dimensional $1/|x-y|^2$ Ising and Potts Models.

Submitted to *J. Stat. Phys.*

ACMS Publication 87-14

W. Arter, A. Bernoff, A. C. Newell

Wavenumber Selection of Convection Rolls in a Box.

To appear *Phys. Lett.*

ACMS Publication 87-15

W. Arter, A. C. Newell

Numerical Simulation of Rayleigh-Bénard Convection in Shallow Tanks.

Phys. of Fluids

ACMS Publication 87-16

Robert S. Maier

Bounds on the Density of States of Random Schrödinger Operators.

To appear *J. Stat. Phys.* **48**, *425 (1987)*

ACMS Publication 87-17

W. G. Faris, R. S. Maier

The Value of a Random Game: The Advantage of Rationality.

J. Comp. Sys. **1**, *235 (1987)*

ACMS Publication 87-18

Robert S. Maier

A Large Deviation Analysis of Dynamic Data Structures.

Submitted to *J. of Algorithms.*

ACMS Publication 87-19

David Rand

Fractal Bifurcation Sets, Renormalisation Strange Sets and their Universal Invariants. To appear in *Proc. R. Soc.*

ACMS Publication 87-20

David Rand

Universality and Renormalisation in Dynamical Systems.

To appear in *New Dir. in Dyn. Sys.*

ACMS Publication 87-21

J. V. Moloney, H. Adachihara, D. W. McLaughlin, A. C. Newell

Fixed Points and Chaotic Dynamics of an Infinite Dimensional Map

To appear in *Chaos, Noise and Fractals,* eds. E.D.R. Pike and L.A. Lugiato

ACMS Publication 87-22

A. J. Bernoff, L. P. Kwok, S. Lichter

Viscous Cross-Waves: An Analytical Treatment

Submitted to *J. Fluid Mech.*

ACMS Publication 87-23

A. J. Bernoff, S. Lichter

Stability of Steady Cross-Waves: Theory and Experiment

Submitted to *Phys. Review A*

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The Dynamics of Patterns: A Survey

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Contour Dynamics for the Euler Equations:
Curvature Controlled Initial Node Placement and Accuracy

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ABSTRACT

We have performed a systematic study of several contour dynamical algorithms for the Euler equations for short times. We have used the Kirchhoff elliptical vortex alone and subject to weak perturbations. We have found that if the initial placement of nodes is such that the internodal distance is proportional to $(curvature)^{-p}$ where $p \approx 1/3$, then errors in short time calculations are minimized. This follows because the node density is invariant in time.

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Two-Particle Billiard System with Arbitrary Mass Ratio

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ABSTRACT

We describe ergodic properties of the system of two hard discs with arbitrary masses moving on the two dimensional torus.

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Bounded Geodesics for the Atiyah-Hitchin Metric

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ABSTRACT

The Atiyah-Hitchin metric has bounded geodesics which describe bound states of a monopole pair.

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Dynamics of Layered Interfaces Arising from Phase Boundaries

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ABSTRACT

The dynamics of a material in two phases is studied in the context of phase-field models based on a Landau-Ginzburg free energy functional. They consist of a system of two nonlinear diffusion equations for the temperature and order parameter. The interface between the two phases is treated as a moving internal layer in two space dimensions, with thickness $0(\epsilon)$, ϵ being a naturally occurring small parameter. Among other things, a dynamical interfacial relation is derived.

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Discontinuity of the Magnetization in the One-Dimensional $1/|x-y|^2$ Ising and Potts Models

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ABSTRACT

We study the nature of the phase transitions in one dimensional q-state Potts models with couplings of the asymptotic form $J_{x,y} \approx const/|x-y|^2$. For translation invariant systems with well defined $lim_{x\to\infty} x^2 J_x = J^+$ (possibly 0 or ∞) we establish: (1) There is no long range order at inverse temperatures β with $\beta J^+ \leq 1$. (2) If $\beta J^+ > q$ then by sufficiently increasing J_1 the spontaneous magnetization $M(\beta J)$ is made positive. (3) In models with $0 < J^+ < \infty$ the magnetization is discontinuous at the transition point (as originally predicted by Thouless), and obeys: $M(\beta_c) \geq 1/\sqrt{\beta_c J^+}$. (4) For Ising $q = 2$ models with $J^+ < \infty$, it is noted that the correlation function decays as $< \sigma_0 \sigma_x > (\beta) \approx c(\beta)/|x-y|^2$ whenever $\beta < \beta_c$. (1)-(3) are deduced from previous percolation results by utilizing the Fortuin Kasteleyn representation, which also yields other results of independent interest relating Potts models with different values of q.

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Wavenumber Selection of Convection Rolls in a Box

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ABSTRACT

The dynamics of two dimensional Rayleigh-Bénard convection rolls are studied in a finite layer with no-slip, fixed temperature upper and lower boundaries and no-slip insulating sidewalls. The dominant mechanism controlling the number of rolls seen in the layer is an instability concentrated near the sidewalls. This mechanism significantly narrows the band of stable wavenumbers although it can take a time comparable to the long (horizontal) diffusion time scale to operate.

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Numerical Simulation of Rayleigh-Bénard Convection in Shallow Tanks

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ABSTRACT

The FLOW3D code, originally designed to simulate industrial heat flow problems, is found to be suitable for studying Rayleigh-Bénard convection in carefully controlled laboratory experiments. The structures of both rolls and defects in fully developed, laminar convection in shallow tanks are described in detail. Simulations at parameters near the threshold for the onset of turbulence in water show a transition between two time-dependent patterns, one roll-like, the other cellular, in which short wavelength instabilities of know type (skew-varicose) are implicated. Evidence for spatio-temporal intermittency s seen although the turbulence is clearly not fully developed.

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Bounds on the Density of States of Random Schrödinger Operators

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ABSTRACT

Bounds are obtained on the unintegrated density of states $\rho(E)$ of random Schrödinger operators $H = -\Delta + V$ acting on $L^2(R^d)$ or $l^2(Z^d)$. In both cases the random potential is

$$V := \sum_{y \epsilon Z^d} V_y \chi(\Lambda(y))$$

in which the $\{V_y\}_{y\epsilon Z^d}$ are IID random variables with density f. The χ denotes indicator function, and in the continuum case the $\{\Lambda(y)\}_{y\epsilon Z^d}$ are cells of unit dimensions centered on $y\epsilon Z^d$. In the finite-difference case $\Lambda(y)$ denotes the site $y\epsilon Z^d$ itself. Under the assumption $f\epsilon L_0^{1+\epsilon}(R)$ it is proven that in the finite-difference case $\rho\epsilon L^\infty(R)$, and that in the $d = 1$ continuum case $\rho\epsilon L^\infty_{loc}(R)$.

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The Value of a Random Game: The Advantage of Rationality

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ABSTRACT

Two players play against each other in a game with payoffs given by a random n by n matrix with mean zero. If one player adopts a uniform, purely random strategy, then his loss is limited by the law of averages to a quantity proportional to $\sqrt{\log n}/\sqrt{n}$. On the other hand, if he plays an optimal strategy his losses will typically be considerably less. Numerical evidence is presented for the following conjecture: the standard deviation of the value of the game is asymptotically proportional to $1/n$. This smaller loss exhibits the advantage of rationality over randomness. The rational player, moreover, tends as $n \to \infty$ to employ a strategy vector that has half its components zero.

ACMS PUB. 87-18

A Large Deviation Analysis of Dynamic Data Structures

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ABSTRACT

I present a probabilistic analysis of simple list structures, serving as implementations of dictionaries, linear lists and priority queues. Under the assumption of equiprobability of histories, *i.e.*, of evolutions considered up to order isomorphism, I show that the integrated space and time costs of a sequence of n supported operations converge as $n \to \infty$ to Gaussian random variables. Their means are asymptotically proportional to n^2 and their standard deviations to $n^{3/2}$. The proof is an application of the theory of large deviations.

This asymptotic behavior reflects the convergence as $n \to \infty$ of the normalized structure sizes to datatype-dependent deterministic functions of time. This unrealistic determinism unfortunately necessitates rejection of the model of equiprobable histories. A paper of Flajolet, Puech and Vuillemin that did not reach this conclusion (*Inform. Sci.* **38** (1986), 121-146) is shown to contain computational errors.

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Fractal Bifurcation Sets, Renormalisation Strange Sets and their Universal Invariants

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ABSTRACT

The traditional renormalisation formalism as invented to study phase transitions and is used in dynamical systems to study the universal properties of the transition to chaos relies upon finding a hyperbolic saddle point for a judiciously chosen transformation of some function space. Then the geometrical and dynamical structure of the saddle point and its stable manifold is used to deduce physically and mathematically interesting consequences. In this paper I want to discuss a more general situation which has a number of interesting applications to dynamical systems and which, I believe, is of even wider interest because it will have applications in other areas. In this generalisation the role of the fixed point is played by a hyperbolic strange set Λ which can be a strange saddle (*e.g.* a horseshoe) or a strange attractor. I call such sets *renormalisation strange sets*. They are applied to deduce the structure and universality of complex fractal structures in parameter space.

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Universality and Renormalisation in Dynamical Systems

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ABSTRACT

The renormalisation group formalism has lead to a number of fruitful developments in our understanding of the "transition to chaos." The best known examples concern the quantitative universality of period-doubling cascades and the breakdown of invariant circles in dissipative and area-preserving maps. This paper is meant to be an introduction to and biased review of these ideas.

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Fixed Points and Chaotic dynamics of an Infinite Dimensional Map

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ABSTRACT

An appealing idea of modern dynamics is that the complicated and apparently stochastic time behavior of large and even infinite-dimensional nonlinear systems is in fact a manifestation of a deterministic flow on a low dimensional chaotic attractor. If the system is indeed low-dimensional, it is natural to ask whether one can identify the physical characteristics such as the spatial structure of those few active modes which dominate the dynamics. Our thesis is that these modes are closely related to and best described in terms of asymptotically robust, multiparameter solutions of the nonlinear governing equations. In this article we present a review of our progress to date in isolating such nonlinear spatial modes and identify their role in inducing chaotic dynamics.

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Viscous Cross-Waves: An Analytical Treatment

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ABSTRACT

Viscous effects on the excitation of cross-waves due to subharmonic resonance are considered using matched asymptotic expansions. The nonlinear Schrödinger equation for inviscid cross-waves near onset is modified by viscous linear damping and detuning. The relative contributions of the free-surface, sidewalls, bottom, and wavemaker viscous boundary layers are computed. In general, viscosity delays the onset to a finite amplitude of forcing and detunes the resonant frequency.

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Stability of Steady Cross-Waves: Theory and Experiment

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ABSTRACT

A bifurcation analysis is performed in the neighborhood of neutral stability for cross-waves as a function of forcing, detuning, and viscous damping. A transition is seen from a subcritical to a supercritical bifurcation at a critical value of the detuning. The predicted hysteretic behavior is observed experimentally. A similarity scaling in the inviscid limit is also predicted. The experimentally observed bifurcation curves agree with this scaling.

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The Dynamics of Patterns: A Survey

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ABSTRACT

A survey of the derivation and properties of the phase and amplitude equations in systems far from equilibrium is given. We study both the near onset and far from onset cases. In the former, both the amplitude and phase are governed by partial differential equations and, for real boundary conditions, the effects of large scale mean drift fields are minimal. The most interesting new developments concern the role that modulational instabilities play when the dominant microscopic structures are travelling or standing waves. In particular, it is shown that localized and strongly disordered behavior can occur. The application of these ideas in the context of convection in binary fluid mixtures is discussed. Far from onset amplitudes are slaved through algebraic equations to the modulus of the phase gradient, but mean drift effects become more important. I discuss the role that defects and focus singularities of the phase equation play in completing one's understanding of the pattern dynamics. In particular, I suggest that the structure and dynamics of defects can be found from singular (particle-like) solutions of the phase equation and illustrate this idea in two cases. First, I find the shape of the dislocation solution in high Prandtl number fluids. Second I compute solutions which describe the breaking of the circular symmetry about a focus singularity in which the focus (umbilicus) is shifted off-center by a dipole shaped mean drift. Finally, building on ideas first proposed by Gollub, McCarriar and Steinman, I suggest a specific mechanism for the onset of turbulence in convecting fluids of low to moderate Prandtl number at Rayleigh numbers of about 4.5 R_c.

The transition involves two features. First, the wavenumber selected by circular patches about sidewall focus singularities approaches the skew varicose instability boundary and the pattern attempts to eliminate rolls through defect nucleation. Second, and crucial to sustaining the time dependence, I show that the advection of wavenumber by mean drift overcomes the stabilizing effect of diffusion and causes the focus singularities to act as sources of wavenumber. It is the continual production of wavenumber and the resulting defect nucleation initiated by skew varicose instabilities which together lead to the turbulent behavior of the pattern.